Gamut Boundary Descriptors using SMGBD and Convex Hull

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1 Introduction

In color science, a device's color gamut refers to the specific range of colors that can be represented or reproduced by the device within a given color space. The Gamut Boundary Descriptor (GBD) is a crucial concept in characterizing and comparing color gamuts. This document provides an in-depth explanation of the mathematical principles and equations employed in a Python implementation that utilizes the Segment Maxima Gamut Boundary Descriptor (SMGBD) technique in conjunction with the Convex Hull method. The implementation focuses on the LAB color space and involves transformations to and from spherical coordinates.

2 Gamut and Gamut Boundary Descriptors (GBD)

The color gamut of a device is the subset of a color space that contains all colors reproducible by that device. The Gamut Boundary Descriptor (GBD) is a technique to mathematically describe the surface enclosing this subset, often using three-dimensional spaces such as LAB, LCH, or RGB.

2.1 LAB Color Space

The LAB color space, also known as CIELAB, is a three-dimensional color space designed to approximate human vision. It is structured as follows:

- L^*: Represents the lightness of the color, ranging from 0 (black) to 100 (white).
- a^*: Represents the position between red/magenta and green, where positive values indicate red/magenta and negative values indicate green.
- b^*: Represents the position between yellow and blue, where positive values indicate yellow and negative values indicate blue.

2.2 Gamut Representation using GBD

The Gamut Boundary Descriptor (GBD) aims to represent the boundary of the gamut by identifying points on the surface of the color space that are most distant from the origin within specific angular segments. This approach facilitates a detailed and accurate representation of the gamut's shape.

2.3 Segment Maxima Gamut Boundary Descriptor (SMGBD)

The Segment Maxima Gamut Boundary Descriptor (SMGBD) technique is a method that divides the spherical color space into discrete segments and identifies the maximum chroma (color intensity) within each segment. The key idea is to find the furthest points in terms of chroma within each angular segment.

3 Mathematical Formulation

3.1 Conversion from RGB to LAB Space

The first step is to convert the image from the RGB color space to the LAB color space. This conversion is performed using the following transformation:

$$\boldsymbol{LAB} = \mathrm{rgb2lab}(\boldsymbol{RGB}) \tag{1}$$

This transformation is based on the CIE standard, which defines how RGB values map to LAB values by considering the human eye's sensitivity to different wavelengths of light.

3.2 Conversion to Spherical Coordinates

The LAB color space is then transformed into spherical coordinates for easier segmentation and boundary description. The transformation is defined as:

$$r = \sqrt{a^2 + b^2} \quad \text{(Chroma)} \tag{2}$$

$$\theta = \arctan 2(b, a) \quad \text{(Azimuthal angle, in degrees)} \tag{3}$$

$$\phi = \arccos\left(\frac{L}{100}\right) \quad \text{(Polar angle, in degrees)} \tag{4}$$

Here, r represents the chroma (color intensity) derived from a^* and b^*, while θ and ϕ correspond to the angular positions within the color space.

3.3 Segmentation of the Spherical Coordinate Space

To create a Gamut Boundary Descriptor, the spherical coordinate space is divided into $m \times n$ segments, where m represents the division in the azimuthal angle θ and n represents the division in the polar angle ϕ. For each pixel in the image, the corresponding spherical coordinates are computed, and the maximum chroma r is recorded in the appropriate segment:

$$\text{aindex} = \min\left(\left\lfloor \frac{\theta}{\frac{360}{m}} \right\rfloor, m-1\right) \tag{5}$$

$$\text{yindex} = \min\left(\left\lfloor \frac{\phi}{\frac{180}{n}} \right\rfloor, n-1\right) \tag{6}$$

The GBD matrix, which stores the maximum chroma values for each segment, is then populated as:

$$\text{GBD}(\text{aindex}, \text{yindex}) = \max\left(\text{GBD}(\text{aindex}, \text{yindex}), r\right) \tag{7}$$

3.4 Reconversion to LAB Space for Visualization

After constructing the GBD, the spherical coordinates are converted back to LAB space for visualization. The conversion is as follows:

$$a = r\cos(\theta) \tag{8}$$

$$b = r\sin(\theta) \tag{9}$$

$$L = 100\cos(\phi) \tag{10}$$

This reconversion allows for the GBD points to be plotted in the LAB color space.

3.5 Convex Hull Calculation

To enclose the gamut points and obtain a continuous representation of the gamut boundary, the Convex Hull method is applied. The Convex Hull is defined as the smallest convex polyhedron that encloses all GBD points:

$$\text{Convex Hull} = \text{ConvexHull}(\text{GBD points}) \tag{11}$$

Mathematically, for a set of points $\{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n\}$, the convex hull is the set of all convex combinations of these points:

$$\text{Convex Hull} = \left\{ \sum_{i=1}^{n} \lambda_i \boldsymbol{x}_i \mid \lambda_i \geq 0, \sum_{i=1}^{n} \lambda_i = 1 \right\} \tag{12}$$

The resulting Convex Hull encloses all the points, forming a closed shape or volume that represents the device's color gamut in LAB space.

4 Implementation

The following Python code implements the above-described procedure using the ‘skimage‘, ‘numpy‘, and ‘scipy‘ libraries.

5 Reference

1. Morovič, J. (2008). Color gamut mapping. John Wiley & Sons.