A method for visualizing the CIELAB space at constant lightness planes

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September 4, 2024

1 Introduction to CIELAB

The CIELAB color space (also known as the CIE 1976 L*a*b* color space) was created by the International Commission on Illumination (CIE) as a perceptually uniform color space. In the CIELAB model, color is represented in a three-dimensional space where:

- L^* represents lightness.
- a^* represents the color axis from green (-) to red (+).
- b^* represents the color axis from blue (-) to yellow (+).

CIELAB is a device-independent color space, meaning that it defines colors in a way that approximates human vision. The lightness value (L^*) runs from 0 to 100, where 0 represents black and 100 represents white. The a^* and b^* axes represent color opponents: green-red and blue-yellow, respectively.

CIELAB has found widespread application in color science because it is designed to be perceptually uniform. A small change in the values of L^*, a^*, and b^* corresponds to an approximately equal perceptual change in color.

2 Overview of method

A method is presented here that is designed to allow a user to explore and interact with the CIELAB color space. Specifically, it allows users to:

1. Specify a lightness value L^*.
2. Interactively click on points in a 2D plot representing the a^* and b^* color axes.
3. The clicked points are annotated with their respective a^* and b^* coordinates.
4. Once a specified number of points is selected, the convex hull is drawn around these points, representing the boundary enclosing these selected colors in the CIELAB space.

3 Significance of the method

This method is significant in that it provides a way for users to visualize the CIELAB color space at a particular lightness level and interact with it. By allowing users to click on the plot and compute a convex hull, it gives a sense of the range of colors present in a specific region of the CIELAB space.

The convex hull function draws the minimal polygon that encloses all the selected points, showing the gamut of selected colors. Such an approach can be used in various color science applications such as analyzing color ranges, optimizing color palettes, or understanding color distributions in a given context.

4 Mathematical Foundations

4.1 CIELAB to RGB Conversion

CIELAB is a color space designed to approximate human vision, but for practical applications like display on a monitor, colors need to be converted to RGB. The RGB system is not perceptually uniform, but it is widely used for rendering colors on screens.

The general steps to convert CIELAB values to RGB involve a series of transformations through the CIEXYZ color space, which is mathematically related to CIELAB. The conversion process is non-trivial and requires several non-linear transformations. Below are the fundamental equations that guide this conversion:

4.1.1 From CIELAB to CIEXYZ

The conversion from CIELAB (L^*, a^*, b^*) to CIEXYZ (X, Y, Z) is defined as follows:

$$
\begin{aligned}
Y &= Y_n \left(\frac{L^* + 16}{116} \right)^3 && \text{if } \left(\frac{L^* + 16}{116} \right) > \delta \\
Y &= Y_n \left(\frac{L^* + 16}{116} \right) / 7.787 && \text{otherwise}
\end{aligned}
$$

$$
\begin{aligned}
X &= X_n \left(\frac{a^*}{500} + f_Y \right)^3 && \text{if } f_Y + \frac{a^*}{500} > \delta \\
X &= X_n \left(\frac{a^*}{500} + f_Y \right) / 7.787 && \text{otherwise}
\end{aligned}
$$

$$
\begin{aligned}
Z &= Z_n \left(f_Y - \frac{b^*}{200} \right)^3 && \text{if } f_Y - \frac{b^*}{200} > \delta \\
Z &= Z_n \left(f_Y - \frac{b^*}{200} \right) / 7.787 && \text{otherwise}
\end{aligned}
$$

Where X_n, Y_n, and Z_n are the white point reference values in the CIEXYZ color space, and δ is a small threshold value.

4.1.2 From CIEXYZ to RGB

Once we have the CIEXYZ values, we can convert them to RGB through a linear transformation matrix:

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 3.2406 & -1.5372 & -0.4986 \\ -0.9689 & 1.8758 & 0.0415 \\ 0.0557 & -0.2040 & 1.0570 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

The RGB values need to be clamped between 0 and 1, and then gamma-corrected for proper display on screens.

4.2 Convex Hull Computation

The convex hull is a geometric construct that represents the smallest convex polygon enclosing a set of points. In this code, once the user has selected the required number of points by clicking on the plot, a convex hull is computed using the `ConvexHull` class from the `scipy.spatial` library.

The algorithm for computing the convex hull is based on the QuickHull algorithm, which is similar to the QuickSort algorithm in terms of complexity and approach.

Mathematically, the convex hull H of a set of points $P = \{p_1, p_2, ..., p_n\}$ is the smallest convex set that contains all points in P.

$$H = \left\{ \sum_{i=1}^{n} \lambda_i p_i \mid \lambda_i \geq 0, \sum_{i=1}^{n} \lambda_i = 1 \right\}$$

This is useful in various fields of computational geometry, graphics, and data analysis, especially when you want to find the outer boundary of a set of points.

5 Conclusion

The presented method allows users to interact with this color space by exploring L^*, a^*, and b^* values and visualizing a convex hull of selected points. It provides an interactive and educational approach to exploring the CIELAB space.