A method for plotting the gamut for multiple chromaticities

Shankhya Debnath

September 3, 2024

1 Introduction

The CIE 1931 chromaticity diagram is a two-dimensional representation of colors as perceived by the human eye. It was developed by the International Commission on Illumination (CIE) in 1931 and has since become a foundational tool in color science. The diagram is based on the human visual system's response to different wavelengths of light and provides a standardized way to quantify and communicate color.

2 The CIE Chromaticity Space

The CIE 1931 chromaticity diagram represents colors based on the tristimulus values X, Y, and Z, which correspond to the color-matching functions derived from experiments with human observers. These values are combined to calculate the chromaticity coordinates x and y, which are defined as:

$$x = \frac{X}{X+Y+Z} \tag{1}$$

$$y = \frac{Y}{X+Y+Z} \tag{2}$$

The x and y coordinates are plotted on the CIE 1931 chromaticity diagram, with each point corresponding to a specific color perceived by the human eye. The diagram encompasses the entire range of visible colors, with the spectral colors (monochromatic light) forming a boundary known as the spectral locus.

3 Applications of the CIE Chromaticity Diagram

The CIE chromaticity diagram is widely used in various fields such as colorimetry, computer graphics, and display technology. It allows for:

- Quantitative comparison of colors.
- Conversion between different color spaces (e.g., RGB, CMYK).
- Assessment of color differences and color matching.
- Design of color calibration systems for displays and printers.

By providing a visual and mathematical framework for color representation, the CIE diagram is essential for ensuring consistent color reproduction across different devices and media.

4 Overview of the Implemented Code

The code utilizes the CIE 1931 chromaticity diagram to allow users to interactively select points within the chromaticity space. Once the user selects a specified number of points, the code computes the convex hull, which is the smallest convex polygon that encloses all selected points, and then displays it on the diagram. This can be useful for applications such as identifying the gamut of colors achievable by a particular display or set of primary colors.

5 Mathematics Behind the Code

5.1 Chromaticity Coordinates Calculation

Given the tristimulus values X, Y, and Z, the chromaticity coordinates x and y are calculated using equations (1) and (2) as mentioned earlier. These coordinates are plotted on the CIE chromaticity diagram to represent colors.

5.2 Conversion to RGB Space

To visualize the colors within the chromaticity diagram on a standard display, the chromaticity coordinates are converted to the RGB color space. This conversion is performed using the following matrix multiplication:

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 3.2406 & -1.5372 & -0.4986 \\ -0.9689 & 1.8758 & 0.0415 \\ 0.0557 & -0.2040 & 1.0570 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{3}$$

After the matrix multiplication, the resulting RGB values are normalized and gamma-corrected to ensure that they lie within the range [0, 1], suitable for display on typical screens.

5.3 Convex Hull Calculation

The convex hull is the smallest convex polygon that can enclose a set of points. It is calculated using the Convex Hull algorithm, which identifies the vertices of the polygon that form the outer boundary of the selected points. Mathematically, given a set of points $P = \{p_1, p_2, \ldots, p_n\}$ in the plane, the convex hull $CH(P)$ is the smallest convex set that contains all points in P.

If H is the set of points that forms the convex hull, then:

$$H = \{p_i \in P \mid p_i \text{ is a vertex of the convex hull}\} \tag{4}$$

The convex hull is then plotted on the CIE chromaticity diagram, illustrating the boundary within which all the selected points lie.

6 Conclusion

The interactive code discussed here allows users to explore the chromaticity space, select points, and visualize the convex hull that encompasses those points. By combining the principles of color science and computational geometry, this tool provides valuable insights into the properties and relationships of colors within the chromaticity space.