A sigmoidal gamut compression algorithm

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1 Introduction

The provided method performs image gamut compression by transforming the image from the RGB color space to the CIELAB color space. It then compresses the lightness and chroma components of the colors to ensure that the image colors fit within the CMYK printer gamut. The results are visualized by plotting the original and compressed image gamuts along with the CMYK printer gamut in the CIELAB space.

2 Mathematical Concepts Involved

The key mathematical concepts involved in the code are:

- **Color Space Conversion**: Conversion between the RGB and CIELAB color spaces.
- **Hue, Chroma, and Lightness Calculation**: Decomposition of the Lab color space into Lightness (L*), Chroma (C*), and Hue (h).
- **Sigmoidal Compression**: A non-linear compression of the lightness and chroma values to bring the colors within the CMYK gamut.
- **Convex Hull Calculation**: A convex hull is computed for each gamut to represent the outer boundary of the color space in three dimensions.

3 Color Space Conversion: RGB to CIELAB

The conversion from RGB to CIELAB is a two-step process:

3.1 RGB to XYZ Conversion

The RGB color values (denoted as R, G, and B) are first normalized and then converted to the XYZ color space using a transformation matrix, which depends on the RGB color space (typically sRGB in this case). The XYZ color components are calculated as:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 0.4124564 & 0.3575761 & 0.1804375 \\ 0.2126729 & 0.7151522 & 0.0721750 \\ 0.0193339 & 0.1191920 & 0.9503041 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

3.2 XYZ to CIELAB Conversion

The XYZ color components are then converted to the CIELAB color space using non-linear transformations. Let X_n, Y_n, and Z_n represent the reference white values. The CIELAB components are calculated as:

$$L^* = 116 \cdot f\left(\frac{Y}{Y_n}\right) - 16$$

$$a^* = 500 \cdot \left(f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right)\right)$$

$$b^* = 200 \cdot \left(f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right)\right)$$

Where the function $f(t)$ is defined as:

$$f(t) = \begin{cases} t^{1/3}, & \text{if } t > 0.008856 \\ 7.787t + \frac{16}{116}, & \text{if } t \leq 0.008856 \end{cases}$$

4 Conversion to LCH (Lightness, Chroma, Hue)

Once the image is converted to CIELAB, the Lab color values are further transformed into the LCH (Lightness, Chroma, Hue) format:

$$C = \sqrt{a^2 + b^2}$$

$$h = \arctan\left(\frac{b}{a}\right)$$

5 Sigmoidal Compression

The compression of lightness and chroma values is done using a sigmoidal function. The sigmoidal compression function applied to both lightness and chroma is given by:

$$x_{\text{norm}} = \frac{2(x - x_{\text{min}})}{x_{\text{max}} - x_{\text{min}}} - 1$$

$$y = \frac{x_{\text{norm}}}{1 + \lambda \cdot |x_{\text{norm}}|}$$

$$y_{\text{mapped}} = \frac{y + 1}{2} \cdot (x_{\text{max}} - x_{\text{min}}) + x_{\text{min}}$$

Where λ is the `compression_strength`, controlling the aggressiveness of the compression. A higher λ results in a stronger compression, forcing more values closer to the middle of the allowable range.

6 Convex Hull and Gamut Calculation

The convex hull is computed for the set of colors in the Lab color space. The convex hull represents the outer boundary of the color gamut in three dimensions. Given a set of points S, the convex hull is the smallest convex set that contains all points in S.

The volumes of the gamuts (original image, compressed image, and CMYK gamut) are calculated using the convex hull. The volume of a convex hull in 3D is given by:

$$\text{Volume} = \frac{1}{3} \sum_{\text{triangles}} \text{Area} \times \text{Height}$$

Where the triangles represent the faces of the convex hull.

7 Explanation of Key Parameters

7.1 Hue Difference (`hue_diff`)

The `hue_diff` variable computes the angular difference between the hue of each color in the image and the corresponding hue in the CMYK gamut:

$$\text{hue\_diff} = \min\left(|\theta_{\text{image}} - \theta_{\text{gamut}}|, 360^\circ - |\theta_{\text{image}} - \theta_{\text{gamut}}|\right)$$

This ensures that the shortest angular distance is used, respecting the cyclical nature of the hue angle.

7.2 Compression Strength (`compression_strength`)

The `compression_strength` parameter λ controls the strength of the sigmoidal compression. Larger values of λ lead to stronger compression, meaning that lightness and chroma values are compressed more toward the middle of their allowable range.

8 Results

After compression, the image colors become less saturated. This is due to the chroma compression, which reduces the intensity of colors to ensure that they fit within the CMYK gamut.

The gamut comparison shows the original image gamut (in grayscale), the compressed image gamut (in color), and the CMYK printer gamut (wireframe).

The compressed image gamut is smaller and fits entirely within the CMYK gamut, ensuring that the image can be printed without out-of-gamut colors.