Radial Basis Function (RBF) Based Gamut Boundary Descriptor (GBD) Method

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1 Introduction

The Radial Basis Function (RBF) based Gamut Boundary Descriptor (GBD) method is a sophisticated technique for approximating and visualizing the color gamut boundary in the CIELAB color space. This paper details the process of using RBF interpolation to create a smooth, continuous surface that represents the gamut boundary, providing an insightful visualization of the color space.

2 CIELAB Color Space Conversion

The process begins by converting an image from the RGB color space to the CIELAB color space, which is more suitable for perceptual analysis and gamut boundary calculations. The conversion is performed as follows:

1. Convert the RGB values to XYZ tristimulus values.
2. Convert XYZ values to CIELAB using the standard equations:

$$L^* = 116 \cdot f\left(\frac{Y}{Y_n}\right) - 16, \tag{1}$$

$$a^* = 500 \cdot \left[f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right) \right], \tag{2}$$

$$b^* = 200 \cdot \left[f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right) \right], \tag{3}$$

where the function $f(t)$ is defined as:

$$f(t) = \begin{cases} t^{1/3} & \text{if } t > \left(\frac{6}{29}\right)^3, \\ \frac{1}{3}\left(\frac{29}{6}\right)^2 t + \frac{4}{29} & \text{otherwise.} \end{cases} \tag{4}$$

3 Radial Basis Function (RBF) Interpolation

Radial Basis Functions (RBF) are a powerful tool for interpolating multi-dimensional data. In this context, RBFs are used to approximate the color gamut boundary by fitting a smooth surface to the scattered LAB color points extracted from the image.

3.1 RBF Formulation

Given a set of n data points $\{\mathbf{x}_i\}_{i=1}^n$ in $\mathbb{R}^3$ (corresponding to LAB values), and corresponding function values $\{L_i\}_{i=1}^n$, the RBF interpolation is expressed as:

$$s(\mathbf{x}) = \sum_{i=1}^{n} \lambda_i \cdot \phi(\|\mathbf{x} - \mathbf{x}_i\|), \tag{5}$$

where:

- $\mathbf{x} = (a^*, b^*)$ are the input coordinates in the a^*b^* plane.
- $\phi(r)$ is the radial basis function, typically chosen as the multiquadric function:

  $$\phi(r) = \sqrt{r^2 + \epsilon^2}, \tag{6}$$

  where $r = \|\mathbf{x} - \mathbf{x}_i\|$ is the Euclidean distance between points, and ϵ is a shape parameter controlling the smoothness.
- λ_i are the weights determined by solving a system of linear equations based on the given data.

3.2 Grid Generation and RBF Fitting

The LAB color points are first downsampled for computational efficiency. The interpolation is then performed over a grid of a^* and b^* values:

1. Create a grid $\mathbf{P}$ in the a^*b^* plane:

   $$\mathbf{P} = \{(a^*, b^*) | a^* \in [a^*_{\text{min}}, a^*_{\text{max}}], b^* \in [b^*_{\text{min}}, b^*_{\text{max}}]\}. \tag{7}$$

2. Fit the RBF model using the available LAB points.
3. Predict the corresponding L^* values for each point on the grid $\mathbf{P}$.

4 3D Visualization of the Gamut Boundary

The interpolated L^* values are used to construct a smooth 3D surface in the LAB color space. The visualization highlights the boundary of the color gamut, providing a clear and intuitive representation of the gamut's structure.

4.1 3D Plotting of the RBF Surface

The 3D surface is plotted using the following steps:

1. Scatter the original LAB points in the 3D space to visualize the distribution of colors.
2. Use the interpolated L^* values to plot a smooth surface over the a^*b^* grid.
3. The surface is colored using the corresponding RGB values, converted back from LAB for accurate representation.

5 Conclusion

The RBF-based GBD method provides a powerful approach to approximating and visualizing the color gamut boundary in the CIELAB color space. By leveraging RBF interpolation, this method generates a smooth, continuous surface that accurately represents the structure of the gamut, offering valuable insights for color analysis and gamut mapping.

6 Reference

1. Park, I. H., Oh, H., & Kang, K. (2019). A simple approach for gamut boundary description using radial basis function network. Electronic Imaging, 31, 1-7.