Gamut Mapping Using Minimum Color Difference Clipping

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1 Introduction

Gamut mapping is the process of transforming the colors of an image from one color space (input gamut) to another (target gamut). This is essential when reproducing images on different devices that may have different color reproduction capabilities. The goal of gamut mapping is to maintain the visual appearance of the image as closely as possible while fitting the colors within the target gamut. In this document, we explore the mathematical foundation of the Minimum Color Difference Gamut Clipping method, which minimizes the perceived color difference when mapping out-of-gamut colors to the nearest boundary of the target gamut.

2 Color Spaces and Gamut Boundaries

2.1 CIELAB Color Space

The CIELAB color space is a perceptually uniform color space, where the distance between two colors corresponds to the perceived difference between them. A color in the CIELAB space is represented by a vector $\mathbf{L}^*\mathbf{a}^*\mathbf{b}^*$, where:

$$\mathbf{L}^* \text{ represents lightness, ranging from 0 (black) to 100 (white),}$$

$$\mathbf{a}^* \text{ represents the green-red axis, and}$$

$$\mathbf{b}^* \text{ represents the blue-yellow axis.}$$

2.2 Spherical Coordinates in CIELAB Space

To facilitate gamut mapping, we convert the CIELAB coordinates to spherical coordinates (r, θ, ϕ) where:

$$r = \sqrt{a^{*2} + b^{*2}}$$

$$\theta = \text{atan2}(b^*, a^*) \quad (\theta \in [0, 360^\circ])$$

$$\phi = \text{acos}\left(\frac{L^*}{100}\right) \quad (\phi \in [0, 180^\circ])$$

These spherical coordinates simplify the representation of colors in the CIELAB space, particularly when analyzing color gamuts and performing transformations.

3 Gamut Boundary Descriptor (GBD)

The Gamut Boundary Descriptor (GBD) is used to define the outer boundary of a color gamut in a given color space. For the target gamut, such as AdobeRGB, the boundary can be obtained using techniques like Convex Hull, which encloses all the colors within the target space.

3.1 Convex Hull

The Convex Hull of a set of points in a Euclidean space is the smallest convex polytope that encloses all the points. Mathematically, if $\mathbf{X} = \{x_1, x_2, \ldots, x_n\}$ represents the set of color points in the target gamut, the Convex Hull $\mathrm{CH}(\mathbf{X})$ is defined as:

$$\mathrm{CH}(\mathbf{X}) = \left\{ \sum_{i=1}^{n} \lambda_i x_i \mid \lambda_i \geq 0, \sum_{i=1}^{n} \lambda_i = 1 \right\}$$

where λ_i are the convex coefficients.

4 Minimum Color Difference Gamut Clipping

4.1 Objective

The objective of the Minimum Color Difference Gamut Clipping is to map an out-of-gamut color $\mathbf{C_{in}}$ to the closest color on the boundary of the target gamut $\mathbf{C_{out}}$ such that the Euclidean distance in the CIELAB space is minimized.

4.2 Mathematical Formulation

Given an out-of-gamut color $\mathbf{C_{in}}$, the goal is to find the point $\mathbf{C_{out}}$ on the GBD such that the Euclidean distance ΔE is minimized:

$$\Delta E = \|\mathbf{C_{in}} - \mathbf{C_{out}}\|$$

where $\mathbf{C_{out}}$ belongs to the Convex Hull of the target gamut.

4.3 Steps in Gamut Mapping

The following steps outline the process:

1. **Check Gamut Membership:** Determine if $\mathbf{C_{in}}$ is within the target gamut by checking if it lies inside the Convex Hull. If so, set $\mathbf{C_{out}} = \mathbf{C_{in}}$.
2. **Find Closest Point on GBD:** If $\mathbf{C_{in}}$ is outside the target gamut, calculate the intersection of the line passing through $\mathbf{C_{in}}$ and the origin with the GBD. For each tessellated triangle on the GBD, do the following:
   - Compute the plane equation for the triangle.
   - Find the intersection of the line with the plane.
   - Check if the intersection point lies within the triangle.
3. **Compute Euclidean Distance:** Calculate the Euclidean distance between $\mathbf{C_{in}}$ and all candidate points on the GBD and select the point with the minimum distance as $\mathbf{C_{out}}$.
4. **Map to Target Space:** Replace the original color $\mathbf{C_{in}}$ with the mapped color $\mathbf{C_{out}}$.

4.4 Tessellation and Surface Intersections

Given the tessellation of the gamut boundary, each tessellated triangle represents a section of the boundary. The intersection of a ray (from the center of the spherical coordinates through the input color) with the triangle's plane can be computed using the parametric form of the plane equation.

The equation of the plane defined by three points $\mathbf{p_1}$, $\mathbf{p_2}$, and $\mathbf{p_3}$ in 3D space is:

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{p_1}) = 0$$

where $\mathbf{n}$ is the normal vector to the plane, $\mathbf{r}$ is the position vector of any point on the plane, and $\mathbf{p_1}$ is a known point on the plane.

The intersection point can be found by solving the system of linear equations obtained from the plane equation and the line equation in parametric form.

5 Shortcomings

The Minimum Color Difference Gamut Clipping method provides an effective approach to mapping colors to a target gamut, however it has its own shortcomings:

- **Reduced Contrast:** The reproduction exhibits noticeably lower contrast compared to the original image.
- **Diminished Colorfulness:** The colors in the reproduced image appear less vibrant and colorful.
- **Loss of Detail:** There is a discernible loss of detail in certain areas of the image.

6 Reference

1. Morovič, J. (2008). Color gamut mapping. John Wiley & Sons.